

Is the Luttinger liquid a new state of matter?

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We are demonstrating that the Luttinger model with short range interaction can be treated as a type of Fermi liquid. In line with the main dogma of Landau's theory one can define a fermion excitation renormalized by interaction and show that in terms of these fermions any excited state of the system is described by free particles. The fermions are a mixture of renormalized right and left electrons. The electric charge and chirality of the Landau quasi-particle is discussed.

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I. INTRODUCTION

The theory of one-dimensional interacting fermions (1D system) has been developed for more than five decades. First time one considered the problem as a simplification of a real 3D task [1] but later it was realized that 1D problems are interesting by themselves (especially for the solid state physics where 1D systems are accessible to a direct measurements). One of the most essential achievements in 1D theory without back scattering was a demonstration of the fact that long wave excitations can be expressed in terms of non-interacting bosons (Tomonaga [2] and Luttinger [3]). In explicit form these expressions were presented by Mattis and Lieb [4]. In the subsequent papers this approach was called "bosonization". Bosonization procedure allows one to proceed from strong interacting 1D electrons to the non-interacting Bose-particles. In other words, the Fermi-type excitations disappear from the theory. It was one of the immediate reasons which led to the conclusion that Fermi liquid theory breaks down in 1D systems. The other one is the form of a Green functions of right (or left) electrons (see e.g. the review paper [5]). Instead of poles they have only a branch cut, i.e. do not correspond to a free fermion excitation. As a result it is commonly believed that the system of 1D interacting electrons is a new state of matter which is not described by the Fermi liquid theory [6] ("non-Fermi liquid state"). "Luttinger liquids (LL) are non-Fermi liquids: Landau quasi-particles are not elementary excitations of the LL and as a consequence the electron Green's function shows no quasiparticle pole..." (citation is taken from the paper [7].) We cannot agree with such argumentation. From our point of view, the form of renormalized electron Green function demonstrates only that the electrons themselves should not be considered as candidates for Landau quasi-particles. Indeed, the renormalized electrons are weak interacting Landau quasi-particles in a 3D normal metal where the Coulomb interaction is sufficiently weak [8]. However, the Landau theory does not require that quasi-particle should coincide with renormalized electron in all cases. According to Landau [9], weak interacting quasi-particles should exist but their origin is not specified. Notice, in a wide sense the term "Fermi-type liquid" was already used in [8] for the normal excitations in 3D superconductor. Here the weakly interacting excitation is a superposition of the electron and hole with opposite momenta and spins [10]; it does not coincide with a renormalized electron. Later on, as a consequence of the Fermi liquid behavior of normal excitations, one has understood that a long range excitation in 3D superconductors can be described by the classical kinetic equation (similar to the Boltzmann equation) [11]. This approach appears extremely successful and enables one to solve numerous problems in superconductivity.

The strongest interaction in the Luttinger model is the interaction between right and left electrons. Hence one can expect that well-defined quasi-particle is a mixture of right and left electrons (c.f. with 3D superconductivity) rather than simple renormalized electron. Reason of the assumption is in absolute necessity to include a strong interaction (changing the ground state of a system) in the initial approximation in order to define a new exact ground state. One cannot consider such interaction perturbatively. Excited states have to be built over the new *stable* ground state. Only such excitations have a chance to be well-defined Landau quasiparticles. Such approach was used by Carmelo and Ovchinnikov for the Hubbard model [12] and Carmelo and Horsch [13] for the Hubbard chain in a magnetic field. Later [14], it was shown that low-temperature thermodynamics of the Hubbard model can be explained in terms of weak interacting quasiparticles that are defined over interacting ground state. This method seems to be more acceptable than examination of a form of renormalized electron Green function (as used here the last is an attempt to build a well-defined quasiparticle over filled 1D Fermi sphere unstable relative to infinitesimal interaction).

We will see below that the analogy with 3D superconductivity is justified and one can introduce noninteracting quasi-particles of such type for Luttinger model with short range interaction. This does not mean that that bosonization procedure is invalid. The system can be described equally well in both representations, either of free bosons or free

fermions. In fact, this should be expected from the very beginning. Indeed, the equivalence of boson and fermion representation for the non-interacting system is well-known. Let us recall the physical reasons of this phenomenon. The fermion-boson equivalence is due to the linearity of the Hamiltonian and one-dimensionality of the system. In such a system it is not possible to distinguish the wave packet composed of a number of the particles from an elementary particle. In other words, there is no way to judge, whether given excitation is elementary or consists of many elementary excitations. Usually one can distinguish one particle state if the total energy of the state is a function only of a total momentum of this state (no internal momenta). However, in the case under consideration this is always true for any number of particles. Mathematically this means that in free one-dimensional system with linear spectrum one can construct a stable packet with virtually any transformational properties and call it elementary excitation. What really matters is the influence of the interaction on the stability of a packet. However it is well-known the effect of short-range interaction in the Luttinger model reduces only to the renormalization of the excitation velocity while the spectrum remains linear. Therefore a wave packet and a single particle do not differ even in the model with interaction. Thus, free fermion and boson descriptions should be possible at the same time. Moreover, even in the model with non-local (but decreasing with distance) e-e interaction, the long wave excitations can be described in fermion language in the region where wavelength is larger than the characteristic size of the interaction. On the other hand, if e-e interaction does not decrease with distance, it is evident that fermion states cannot exist. One calls this as a confinement of electrons. Well-known example is the Schwinger [1] model where the e-e potential is a linear (*increasing!*) function of the distance. In this model the energy spectrum has a gap and our reasoning is not valid. Only boson description is possible in this model.

Can one call the 1D system with short range interaction that is described in terms of noninteracting fermions as a Landau-type liquid? We believe the answer is positive. According to the main principles of the Landau theory [8] one should be able to define a quasi-particles and show that in terms of new fermions the excitations of the system are described by free Hamiltonian. This is exactly the case. As regards the observed values, one can rewrite it in the terms of new quasi-particles.

As appears from the above, it is possible to construct packets with virtually any transformational properties and quantum numbers [7](electric charge, chirality, fermion number, etc) in the interacting theory. Most of them will remain strong interacting. Nevertheless, they can be stable too. However, only weak interacting packets can be considered as final excitations of the interacting theory. In the Luttinger liquid with a short range interaction these are either bosons or fermions with definite quantum numbers. What representation to use is, in a sense, a matter of convenience so far as some additional interaction removes degeneracy of one particle states and the states consisting of many elementary excitations. We know that in the Schwinger model the degeneracy is removed in favor of bosons. This means that fermions are confined and it is due to linear growth of the inter-electron potential. We see no reason for existence of such phenomenon in the ordinary non-relativistic 1D models with decreasing interaction and believe that long range excitations of the system can be described as a Fermi liquid of Landau type.

Finally, let us compare the conception asserting that the Luttinger liquid is a new non-Fermi liquid state and the viewpoint offered in the present paper. The first was formulated e.g. in [15] as a motivation of the work. "The concept of Luttinger liquid is an alternative to the Fermi liquid elaborated for one-dimensional electronic systems. It was found that in 1D electronic systems the Fermi-liquid picture breaks down even in case of arbitrary weak interaction. Single-electron quasiparticles cannot exist in 1D metals, and electrons form the Luttinger liquid in which the only low energy excitations turn out to be charge and spin collective modes with the soundlike spectrum..." According to this viewpoint, the Luttinger liquid segregates from other physical systems and has nothing in common with other 3D electron systems. On the contrary, we believe that the excitations in the Luttinger liquid with a short range interaction can be described as well as a Fermi-liquid state. In this regard the Luttinger liquid is similar to 3D systems. (An immediate analogy is the normal excitations in 3D systems with a long range order, e.g. in superconductors.) At the same time, a significant difference between Luttinger liquid and 3D system exists. First, because of one-dimensionality, in the Luttinger liquid with a short range interaction the free fermion and free boson representations coexist. Secondly, owing to strong e-e interaction relation between Landau quasi-particles and renormalized right and left electrons (similar to Bogolubov-Valantin transformation [10] in 3D superconductivity) is non-linear, etc. However, there are details. In general excited states of the systems qualitatively resemble. (Existence common features of 1D Hubbard chain and 3D Fermi liquids has been indicated in [16].)

At the same time resemblance does not mean identity. In the LL model the electron fluxes differ from quasiparticle ones. It is a usual situation for a system with a long range order because in this case a quasiparticle consists of a mixture of different electron states. So, if one expresses one flux through other extra factors appear. (In 3D superconductivity one calls them "coherence factors.") They change the electron transport coefficients (defined by normal excitations) in comparison with ordinary expressions for the conventional Landau liquid. For Luttinger liquid the difference is even more essential because relation between creation operator of an electron and the quasiparticle operators is nonlinear (see below). So, physical properties of the system can differ essentially from a conventional Fermi system. However, the crucial point for the Fermi liquid theory is existence well-defined quasiparticles and this

is the case. In this regard one may treat excitations of the Luttinger liquid (as well as normal excitations in other systems with long range order) as a generalized Landau Fermi liquid. The main message of the paper is: "For a short range e-e interaction the conceptions of the Luttinger and Landau liquids do not contradict one another but coexist."

The last point has to be considered in the paper is the relation between fermionization of the Luttinger liquid and the Dzyaloshinskii-Larkin theorem [17]. According the theorem Random Phase Approximation approach (RPA) is exact for the model with short range interaction. We discuss the problem in the Appendix B.

II. FERMION REPRESENTATION OF THE EXCITATION.

A. Spinless fermion

In order to introduce a notation and discuss a point which is crucial for further discussion we begin with a brief review of the well-known bosonization procedure. As usual, we divide the electron wave function $\hat{\Psi}(x)$ into wave functions of the left- and right- particles ($\hat{\Psi}_{L,R}$ respectively):

$$\hat{\Psi}(x) = \exp(ip_f x) \hat{\Psi}_R(x) + \exp(-ip_f x) \hat{\Psi}_L(x), \quad (1)$$

here p_f is the Fermi momentum. Throughout the paper we are interested in a model with short-range electron-electron interaction:

$$H = \int dx \left[\hat{\Psi}_R^\dagger(x) v_f (-i\partial_x) \hat{\Psi}_R(x) + \hat{\Psi}_L^\dagger(x) v_f i\partial_x \hat{\Psi}_L(x) \right] + \int dx \hat{\rho}(x) V_0 \hat{\rho}(x), \quad (2)$$

where $\hat{\rho}(x)$ is the operator of the total electron density:

$$\hat{\rho}(x) = \hat{\rho}_R(x) + \hat{\rho}_L(x).$$

Note that the densities of the left- and right-particles, $\hat{\rho}_{R,L}$, obey the well-known Schwinger anomaly [1]:

$$[\hat{\rho}_{R,L}(x), \hat{\rho}_{R,L}(y)] = \pm \frac{i}{2\pi} \frac{\partial}{\partial x} \delta(x-y). \quad (3)$$

The last equation is a starting point for bosonization procedure [4]. One introduce two Bose fields corresponding to the left- and right-electron densities:

$$\hat{C}_{R,L}(p) = \sqrt{\frac{2\pi}{p}} \int dx \exp(\mp ipx) \hat{\rho}_{R,L}(x). \quad (4)$$

So, fields $\hat{C}_{R,L}, \hat{C}_{R,L}^\dagger$ commute in the way usual for Bose-particles (in Eqs. (4),(5) all $p > 0$). The following transformation

$$\hat{C}(p) = \cosh \theta \hat{C}_R(p) + \sinh \theta \hat{C}_L^\dagger(p) \quad (5)$$

$$\hat{C}^\dagger(-p) = \cosh \theta \hat{C}_L^\dagger(p) + \sinh \theta \hat{C}_R(p)$$

allows to diagonalize the Hamiltonian if θ is chosen in such a way that $\sinh 2\theta = V_0/2\pi v_f^c$ (which retains the commutator of the operators):

$$H = v_f^c \int_0^\infty \frac{dp}{2\pi} p \left(\hat{C}^\dagger(p) \hat{C}(p) + \hat{C}^\dagger(-p) \hat{C}(-p) \right). \quad (6)$$

Here

$$v_f^c = v_f \sqrt{1 + \frac{V_0}{\pi v_f}}$$

is the renormalized Fermi velocity (see, for example [18]). It is convenient for as to use the bosonization scheme operating with particles $\hat{C}_{L,R}$ (see [19]) rather than a scheme with a total electron density and momentum canonically conjugate to it. It is possible to present Ψ in terms of boson operators as follows:

$$\Psi_{R,L}^\dagger(x) = \exp\left(\mathcal{A}_{R,L}^\dagger(x)\right) \frac{\hat{\sigma}_{R,L}^\dagger}{\sqrt{L}} \exp(-\mathcal{A}_{R,L}(x)). \quad (7)$$

Here

$$\mathcal{A}_{R,L}^\dagger(x) = \int_{2\pi/L}^\infty \frac{dp}{2\pi} \exp(\mp ipx) \sqrt{\frac{2\pi}{p}} \hat{C}_{R,L}^\dagger(p),$$

while $\hat{\sigma}$ is the operator similar to the ladder operator introduced by Haldane [20]. In order to have correct commutation rules for the $\Psi_{R,L}$ one should require: $\hat{\sigma}_{R,L}^\dagger \hat{\sigma}_{R,L} = 1$, $\{\hat{\sigma}_{R,L}, \hat{\sigma}_{L,R}\} = 0$. Besides, one can see that $\hat{\sigma}$ and $\hat{\sigma}^\dagger$ commute with $\hat{C}_{L,R}$.

However, mathematically the exact Hamiltonian (Eq.6) differs from the boson Hamiltonian originated of free electrons only by the replacements $v_f^c \rightarrow v_f$ and $\hat{C}(p) \rightarrow \hat{C}_{R,L}(p)$. So, if one defines *fermion* operators ($\hat{\chi}$) coinciding with original R and L -electrons in the limit $V_0 \rightarrow 0$:

$$\hat{\chi}_\pm^\dagger(x) = \exp\left(A_\pm^\dagger(x)\right) \frac{\hat{\sigma}_\pm^\dagger}{\sqrt{L}} \exp(-A_\pm(x)) \quad (8)$$

with

$$A_\pm^\dagger(x) = \int_{2\pi/L}^\infty \frac{dp}{2\pi} \exp(\mp ipx) \sqrt{\frac{2\pi}{p}} \hat{C}^\dagger(\pm p),$$

then one can see directly

$$\{\hat{\chi}_\pm^\dagger(x), \hat{\chi}_\pm(x_1)\} = \delta(x - x_1) \quad (9)$$

(for $|x - x_1| \ll L$) and show that in terms of the χ -particles our system is described by the free Hamiltonian without interaction. (In fact, this is the transformation an electron kinetic energy from fermion to boson representation with replacements noted above. See, for example, [4].) Hence, the fermions are nothing more than original electrons excitation moving together with ground state polarization:

$$H = v_f^c \int dx \left(\hat{\chi}_+^\dagger(x) (-i\partial_x) \hat{\chi}_+(x) + \hat{\chi}_-^\dagger(x) (i\partial_x) \hat{\chi}_-(x) \right). \quad (10)$$

Formally it is shown in Appendix A, there one can see that it is right only for the short-range interaction (v_f^c does not depend on p). Otherwise, four - fermions terms should be added to Eq.(10) [21].

Both Hamiltonians, Eq. (6) and Eq.(10), describe all excited states of the system but in different representations. It is important that the ground state wave function which is well-known in terms of bosons is, in fact, the vacuum state for excitations in both representations. In order to prove that the state with lowest energy (see, for example, [20])

$$|GS_0\rangle = N \exp\left(-\int_0^\infty \frac{dp}{2\pi} \tanh\theta \hat{C}_L^\dagger(p) \hat{C}_R^\dagger(p)\right) |F\rangle \quad (11)$$

is the vacuum state for renormalized field $\hat{\chi}$ one should proceed from whole fermion fields $\hat{\chi}$ to the particle - hole representation ($\mathbf{a}_\pm(x), \mathbf{b}_\pm(x)$):

$$\hat{\chi}_\pm(x) = \int_0^\infty \frac{dp}{2\pi} \left(\exp(\pm ipx) \hat{\mathbf{a}}_\pm(p) + \exp(\mp ipx) \hat{\mathbf{b}}_\pm^\dagger(p) \right) = \hat{\mathbf{a}}_\pm(x) + \hat{\mathbf{b}}_\pm^\dagger(x). \quad (12)$$

(N in Eq.(11) is the normalization coefficient.) So, the electron part can be extracted from $\hat{\chi}$ in the following way:

$$\mathbf{a}_+(y) = \frac{1}{2\pi i} \int dx \frac{\hat{\chi}_+(x)}{x - y - i\delta}. \quad (13)$$

Hence

$$\mathbf{a}_+(y) |GS_0\rangle = \hat{\sigma}_+ \frac{1}{2\pi i \sqrt{L}} \int \frac{dx}{x - y - i\delta} \exp\left(-\int_{2\pi/L}^\infty \frac{dp}{2\pi} \exp(-ipx) \sqrt{\frac{2\pi}{p}} \hat{C}^\dagger(p)\right) |GS_0\rangle = 0, \quad (14)$$

because $\hat{C}(p)|GS_0\rangle = 0$ and the integrand of Eq.(14) has no singularities in lower semiplane. One can prove the same property for the hole part as well. So, in spite of the fact that the ground state Eq.(11) is not empty, the

\hat{a} -particles are not present there. That is, one can consider the particles \hat{a}, \hat{b} as excitations over the ground state Eq.(11). (This requirement is necessary because a ground state is the state with smallest energy, i.e. it is the state without excitations.) Notice, this claim does not hold for an $R(L)$ -electron because $\hat{C}_{R,L}(p)|GS_0\rangle \neq 0$. It means that either the $R(L)$ -electron cannot be considered as an excitation of LL or the ground state Eq.(11) is defined incorrectly. We believe this is the first case.

For larger temperatures $2\pi v_f^c/L \gg T \gg 2\pi v_f/L = T_{chiral}$ the states with different chiralities are degenerate (we have in mind a strong repulsive e-e interaction here), because the $2\pi v_f/L$ is the characteristic energy difference between the states. (Note, the temperature region $2\pi v_f^c/L \gg T$ is the region where power - low correlators exist. At the same time, the state Eq.(11) is the state with zero chirality.) So, the real ground state for the considered temperature region is a mixture of the states with all chiralities and, as a result, it is the state with broken chiral symmetry:

$$|\theta\rangle = \sum_{-\infty}^{\infty} \exp(in\theta) |GS_n\rangle, \quad (15)$$

here

$$|GS_n\rangle = (\hat{\sigma}_L \hat{\sigma}_R^\dagger)^n |GS_0\rangle \quad \text{for } n > 0; \quad |GS_n\rangle = (\hat{\sigma}_R \hat{\sigma}_L^\dagger)^n |GS_0\rangle \quad \text{for } n < 0.$$

(See [22] for detailed discussion.) It is obvious that this state is the vacuum state for both representations too. So, as it should be, well-defined quasi-particle excitations can be obtained only over the stable exact ground state, and not over a filled 1D Fermi sphere. The last state is unstable relative to infinitesimal e-e interaction.

Let us discuss such characteristics of the renormalized fermion $\hat{\chi}$ as electric charge and chirality. We assign chirality +1 to a right electron, and a left hole and -1 to their counterparts. The corresponding "density" is $\hat{\Sigma}(x) = \hat{\rho}_R(x) - \hat{\rho}_L(x)$. So, the last charge is the charge determining contribution to the electric current from a renormalized fermion.

For a system of weak interacting electrons the above question is trivial. In a conventional Fermi liquid the electric charge of a quasi-particle equals to the bare electron one. Here we imply, as usual, that the screening length is a macroscopic one. Then one can measure a charge at the distances smaller than the screening length. In a system with a strong coupling, screening, total or partial, can be realized at the distances smaller than a minimal scale of the theory. A well-known example is the screening of a bare electron charge in quantum electrodynamics. It is also the case for our problem because in the main order in V_0^{-1} the electron system is polarized so strongly that the ground state eq.(11,15), in fact, consists of exciton-like neutral pairs [22] (we have in mind a repulsive interaction here). In other words, the "screening scale" is about the transverse size of the channel. This scale is considered to be zero by an effective 1D theory. In this paper we will see that a similar screening takes place also for the fermion excitations.

In order to define the electric charge of a new fermion we will use the following relations (see, for example, [23])

$$e_0 [\hat{\rho}(x), \hat{\chi}_+(y)]_- = -e^* \delta(x-y) \hat{\chi}_+(y). \quad (16)$$

Here e_0 and e^* are the charges of the bare electron and exact fermion field, respectively. Similar equation is valid for the chirality if one substitutes $\hat{\Sigma}$ in place of $\hat{\rho}$. During the calculation it is convenient to express the density and chiral operators from LHS of Eq.(16) in the form:

$$\hat{\rho}(x) = \int_{-\infty}^{\infty} \frac{dp}{2\pi} \sqrt{\frac{|p|\gamma}{2\pi}} (\exp(ipx) C(p) + \exp(-ipx) C^\dagger(p)) \quad (17)$$

$$\hat{\Sigma}(x) = \int_{-\infty}^{\infty} \frac{dp}{2\pi} \sqrt{\frac{|p|}{2\pi\gamma}} \text{sign}(p) (\exp(ipx) C(p) + \exp(-ipx) C^\dagger(p)),$$

($\gamma = v_f/v_f^c = \exp(-2\theta)$.) Direct calculation gives

$$e^* = \sqrt{\gamma} e_0$$

and

$$[\hat{\Sigma}(x), \hat{\chi}_+(y)]_- = -\frac{1}{\sqrt{\gamma}} \delta(x-y) \hat{\chi}_+(y), \quad (18)$$

i.e. the chiral charge, σ (corresponding to the operator $\hat{\Sigma}$) equal to $1/\sqrt{\gamma}$. Therefore, exact field $\hat{\chi}_+$ is not simply screening R - electron but rather a combination of left- and -right electrons (see relations just below and the next section). Such a relation between the electric charge and chirality allows one to have a finite value of the electric current flowing through the channel (due to an excitation), even for the strong interaction case ($\gamma \rightarrow 0$).

In order to calculate an observable data one should establish connections between operators χ and Ψ . The direct relation

$$\hat{\chi}_+(x) \propto \hat{\sigma}_+ [\hat{\sigma}_R^\dagger \hat{\Psi}_R(x)]^{\cosh \theta} [\hat{\sigma}_L \hat{\Psi}_L^\dagger(x)]^{\sinh \theta}$$

(and a similar one for $\hat{\chi}_-$) in principle gives an opportunity to express one matrix element through another. (We do not write out in full c-number factor here.) However, the self-consistent equation (A2) together with Eqs.(4, 5) give a more convenient way for calculation some of observed quantities:

$$\hat{\varrho}_+(x) = \cosh \theta \hat{\varrho}_R(x) + \sinh \theta \hat{\varrho}_L(x).$$

Similar relations can be written for other bilinear in $\Psi_{R,L}$ operators.

B. Two-component fermion.

For a simplicity in this section we will be interested in most symmetrical version of e-e interaction [18]:

$$H_{int} = 1/2 \sum_{\alpha, \beta=1,2} \int dx \varrho_\alpha(x) V_0 \varrho_\beta(x). \quad (19)$$

As usual, one defines Bose - fields $\hat{C}_{R,L,i}(p)$ similarly to Eq.(4) (with an additional spin index $i = 1, 2$) and separates spin and space variables:

$$\hat{C}_{R,L}(p) = 1/\sqrt{2} \left(\hat{C}_{R,L,1}(p) + \hat{C}_{R,L,2}(p) \right); \hat{S}_{R,L}(p) = 1/\sqrt{2} \left(\hat{C}_{R,L,1}(p) - \hat{C}_{R,L,2}(p) \right). \quad (20)$$

In the case the spin variable separates and can be described by free Hamiltonian with non-perturbed Fermi velocity [18]. Therefore fermionization of this part of the Hamiltonian is trivial (see above). The spinless part of the Hamiltonian can be diagonalized by the same transformation as previously, Eq.(5), but with new fields $\hat{C}_{R,L}(p)$ (Eq.(20)) and the same rotation angle $\exp(-2\theta) = v_f/V_f^c$ where the expression for renormalized velocity should be slightly modified:

$$V_f^c = v_f \sqrt{1 + \frac{2V_0}{\pi v_f}}.$$

As a result, the free-fermion Hamiltonian (similar to Eq.(10)) will depend on V_f^c if we define the new Fermi-fields $\hat{\Xi}_\pm(x)$ as above:

$$\hat{\Xi}_\pm^\dagger(x) = \exp \left(A_\pm^\dagger(x) \right) \frac{\hat{\sigma}_\pm^\dagger}{\sqrt{L}} \exp \left(-A_\pm(x) \right). \quad (21)$$

Let us note that the field $\hat{\Xi}_+^\dagger$ does not turn into the right-electron if the interaction is switched off: $V_0 \rightarrow 0$.

Had one considered the other fields $\hat{\Xi}_{\pm,i}^\dagger$ transferring to $\Psi_{R,i}^\dagger$ in the limit $V_0 \rightarrow 0$:

$$\hat{\Xi}_{\pm,i}^\dagger(x) = \exp \left(\tilde{A}_{\pm,i}^\dagger(x) \right) \frac{\hat{\sigma}_\pm^\dagger}{\sqrt{L}} \exp \left(-\tilde{A}_{\pm,i}(x) \right) \quad (22)$$

with

$$\tilde{A}_{\pm,i}^\dagger(x) = \int_{2\pi/L}^\infty \frac{dp}{2\pi} \sqrt{\frac{2\pi}{p}} \exp(\mp i p x) \frac{1}{\sqrt{2}} \left(\hat{C}^\dagger(\pm p) \pm S_{R,L}^\dagger(p) \right)$$

one would obtain a theory with strong interaction. Different components of the left- (or right -) particles would interact one with other via the vertex which is proportional to $V_f^c - v_f$. Thus, as Fermi-field $\hat{\Xi}$ diagonalizing Hamiltonian does not turn into L or R free electrons when interaction is switched off, it cannot be interpreted as a renormalized electron. Instead one should talk about diagonalizing transformation similar to Bogolubov-Valantin one [10] in the theory of 3D superconductivity.

Just as above one can define the charges of the fermion field $\hat{\Xi}_+$. They are equal to $e^* = \sqrt{2\gamma}e_0$ and $\sigma = 1/\sqrt{2\gamma}$ with $\gamma = v_f/V_f^c$.

III. CONCLUSION

Using free fermion representation for Luttinger model with short-range interaction we extend concept of the Fermi liquid Landau to the system of 1D interacting fermions. The quantum numbers of Landau quasi-particles are different in comparison with original electrons due to the strong polarization of the ground state. In particular, in the limit of infinitely strong repulsive interaction the electric charge of the quasi-particle is screened out completely. At the same time chirality of the quasi-particle in this limit rises steeply, so that their product is equal to free electron one. These quasi-particles describe all excited state of the Luttinger liquid and are an analogy with the normal excitations of Bogolubov-Valantin in the theory of 3D superconductivity.

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Appendix A: From boson to fermion representation.

In order to prove that in χ representation the system is described by a free Hamiltonian one has to repeat the bosonization procedure in the reverse order (see, for example, [4],[19], [20]). First of all, we associate the density of diagonalizing Fermi-fields ($\hat{\rho}_{\pm}$) with the diagonalizing boson fields $\hat{C}(\pm p)$. As usual we define the density of the exact electron as

$$\hat{\rho}_{\pm}(x) = \frac{1}{2} \lim_{\epsilon \rightarrow 0} \left(\hat{\chi}_{\pm}^{\dagger}(x + \epsilon/2) \hat{\chi}_{\pm}(x - \epsilon/2) + \hat{\chi}_{\pm}^{\dagger}(x - \epsilon/2) \hat{\chi}_{\pm}(x + \epsilon/2) \right). \quad (\text{A1})$$

We have to use this definition because the quantity $\hat{\chi}_{\pm}^{\dagger}(x) \hat{\chi}_{\pm}(y)$ is singular at the point $x \rightarrow y$. The singularity has to be redefined in such a way that its Fourier transform $\hat{\rho}_{\pm}(p)$ would be determined by $\hat{C}(p)$ for $p > 0$ and $\hat{C}^{\dagger}(-p)$ for $p < 0$ in the usual way. Definition (A1) satisfies this requirement. Indeed, substituting Eq.(8) in Eq.(A1) one gets:

$$\hat{\rho}_{+}(x) = \int_0^{\infty} \frac{dp}{2\pi} \sqrt{\frac{p}{2\pi}} \left(\hat{C}(p) \exp(ipx) + \hat{C}^{\dagger}(p) \exp(-ipx) \right) \quad (\text{A2})$$

in accordance with analogous equation for R -electron density (4). Using Eq.(A2) and a similar one for $\hat{\rho}_{-}(x)$ one can rewrite Hamiltonian Eq.(6) in the form:

$$H = v_f^c \int_0^{\infty} dp \int dxdy [\hat{\rho}_{+}(x) \hat{\rho}_{+}(y) \exp(ip(x-y)) + \hat{\rho}_{-}(x) \hat{\rho}_{-}(y) \exp(ip(y-x))] \quad (\text{A3})$$

For further calculation it is essential that electron (or hole) part of the fermion operator $\hat{\chi}$ is an operator with simple analytical properties. For example, $\mathbf{b}_{+}^{\dagger}(x)$ has no singularity in the lower semiplane of the complex x , as one can see from definition Eq. (12), etc. Let us discuss first the term $\mathbf{a}_{+}^{\dagger}(x) \mathbf{a}_{+}(x) \mathbf{a}_{+}^{\dagger}(y) \mathbf{a}_{+}(y)$ which arises from the product of two densities $\hat{\rho}_{+}$, where $\hat{\rho}_{+} = \mathbf{a}_{+}^{\dagger} \mathbf{a}_{+} - \mathbf{b}_{+}^{\dagger} \mathbf{b}_{+} + \mathbf{a}_{+}^{\dagger} \mathbf{b}_{+}^{\dagger} + \mathbf{b}_{+} \mathbf{a}_{+}$. We present it in the form:

$$1/2 \int dxdy \int_0^{\infty} dp \left[\mathbf{a}_{+}^{\dagger}(x) \mathbf{a}_{+}(x) \mathbf{a}_{+}^{\dagger}(y) \mathbf{a}_{+}(y) \exp(ip(x-y+i\delta)) + \mathbf{a}_{+}^{\dagger}(y) \mathbf{a}_{+}(y) \mathbf{a}_{+}^{\dagger}(x) \mathbf{a}_{+}(x) \exp(ip(y-x+i\delta)) \right].$$

Then, one should exchange two central operators $\mathbf{a}_{+} \mathbf{a}_{+}^{\dagger}$ in each term using the anticommutator:

$$\{\mathbf{a}_{+}^{\dagger}(x), \mathbf{a}_{+}(y)\} = \frac{1}{2\pi i} \cdot \frac{1}{x-y-i\delta}.$$

The term with four operators vanishes after integration over p (in accordance with the Pauli principle and such is the case only for short range e-e interaction) while the term with two operators after the same integration gives

$$-\frac{1}{2\pi} \int dxdy \mathbf{a}_{+}^{\dagger}(x) \cdot \frac{1}{(y-x-i\delta)^2} \mathbf{a}_{+}(y).$$

An analytical properties of $\mathbf{a}_+(y)$ allow one to present this term in the form

$$\frac{1}{i} \int dx \mathbf{a}_+^\dagger(x) \partial_x \mathbf{a}_+(x),$$

as it should be in accordance with Eq.(10). Each term from (A3) can be considered in the similar way. As a result one would obtain the whole Hamiltonian Eq.(10).

Appendix B: Dzyaloshinskii - Larkin approach and fermionization.

In the appendix we will discuss relation between representation of the Luttinger model by free fermions $\hat{\chi}_\pm$ and Dzyaloshinskii-Larkin approach. In their paper [17] they proved that RPA approach is exact for the Luttinger liquid with short range interaction. From the outset it is clear that free fermions in an RPA bubble and $\hat{\chi}_\pm$ are not the same: they have different velocity, v_f and v_f^c respectively. In order to examine interconnection of the fermions more deeply it is useful to rederive the Dzyaloshinskii-Larkin result in another way.

The theory with arbitrary electron-electron interaction can be reduced to a noninteracting fermion theory in an external field by means of the well-known Hubbard-Stratonovich transformation [24]:

$$\exp \left[-\frac{i}{2} \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} \frac{dp}{2\pi} V(p) \varrho(p, t) \varrho(-p, t) \right] = \frac{1}{\mathcal{N}} \int \mathcal{D}\Phi \exp \left[\frac{i}{2} \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} \frac{dp}{2\pi} \Phi(p, t) \Phi(-p, t) V^{-1}(p) - \frac{i}{2} \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} \frac{dp}{2\pi} (\varrho(p, t) \Phi(-p, t) + \varrho(-p, t) \Phi(p, t)) \right]$$

Here \mathcal{N} is normalization coefficient:

$$\mathcal{N} = \int \mathcal{D}\Phi \exp \left[\frac{i}{2} \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} \frac{dp}{2\pi} \Phi(p, t) \Phi(-p, t) V^{-1}(p) \right],$$

$V(p)$ is Fourier transformation of e-e interaction, and $\int \mathcal{D}\Phi$ is a functional integral over the Bose - fields $\Phi(x, t)$. The transformation allows one to reduce any electron Green function $G_{cul}(x_1, x_2, \dots, x'_1 \dots)$ to similar connected Green function of noninteracting electrons in external electric field $\Phi(x, t)$, $G(\Phi, x_1, x_2, \dots)$:

$$G_{cul}(x_1, \dots, x'_1 \dots) = \int \mathcal{D}\Phi G(\Phi, x_1, \dots, x'_1 \dots) \exp \left(S_N(\Phi) + \log \text{Det} \hat{S}(\Phi) \right) / \int \mathcal{D}\Phi \exp \left(S_N(\Phi) + \log \text{Det} \hat{S}(\Phi) \right), \quad (\text{B1})$$

here

$$S_N(\Phi) = \frac{i}{2} \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} \frac{dp}{2\pi} \Phi(p, t) \Phi(-p, t) V^{-1}(p)$$

is the phase arising from Hubbard-Stratonovich transformation (the first term in the RHS of the transformation) while the operator $\hat{S}(\Phi)$ is defined by the kernel of the electron action in an external field, $\text{Tr} (\bar{\Psi} \hat{S}(\Phi) \Psi)$. During derivation of Eq.(B1) we have "calculated" the functional integral over the electron fields:

$$\int \mathcal{D}\Psi \mathcal{D}\bar{\Psi} \bar{\Psi}(x_1) \dots \Psi(x'_1) \dots \exp \text{Tr} (\bar{\Psi} \hat{S}(\Phi) \Psi) = G(\Phi, x_1, \dots, x'_1 \dots) \exp \left(\log \text{Det} \hat{S}(\Phi) \right) \quad (\text{B2})$$

It is important that in order to calculate a Green function one has to integrate over the fields $\Psi(x, t)$ decaying in the limit $t \rightarrow \pm\infty$. The LHS of Eq.(B2) is the disconnected electron Green function in an external field (with all electron loops). So, $\log \text{Det} \hat{S}(\Phi)$ is the sum of all connected electron loops in an external field. During the calculation of the quantity one has to take into account an ultraviolet divergence. One has to regularize it in a usual way demanding a gauge-invariance of the theory. For the Luttinger spinless fermion model it has been calculated in [22] and equal

$$\begin{aligned} \log \text{Det} \hat{S}(\Phi) = & -\frac{1}{4\pi} \int_{-\infty}^{\infty} dt dt_1 \int_{-\infty}^{\infty} \frac{dp}{2\pi} \Phi(-p, t) \Phi(p, t_1) |p| \exp[-i|p|v_f|t-t_1|] = \\ & -\frac{i}{2\pi} \int_{-\infty}^{\infty} \frac{dp d\omega}{(2\pi)^2} \Phi(p, \omega) \Phi(-p, -\omega) \frac{p^2 v_f}{\omega^2 - p^2 v_f^2 + i\delta} \end{aligned} \quad (\text{B3})$$

Notice, the eq.(B3) is gauge invariant: a field depending only on time does not contribute. So, the phase in Eq.(B1) is equal

$$S_N(\Phi) + \log \text{Det} \hat{S}(\Phi) = \frac{i}{2} \int_{-\infty}^{\infty} \frac{dp d\omega}{(2\pi)^2} \Phi(p, \omega) \Phi(-p, -\omega) V_0(p)^{-1} \frac{\omega^2 - (pv_f^c)^2 + i\delta}{\omega^2 - p^2 v_f^2 + i\delta} \quad (\text{B4})$$

In order to check Eqs.(B1, B4) let us calculate one particle Green function. For the case

$$G_{R,L}(\Phi, x_1, t_1, x_2, t_2) = G_{R,L}^0(x_1, t_1; x_2, t_2) \exp \left[-i \int_{-\infty}^{\infty} dt' \int dy \Phi(y, t') (G_{R,L}^0(x_1, t_1; y, t') - G_{R,L}^0(x_2, t_2; y, t')) \right], \quad (\text{B5})$$

where $G_{R,L}^0$ is a free fermion Green function equal to

$$G_{R,L}^0(x_1, t_1; x_2, t_2) = \frac{1}{2\pi i} [v_f(t_1 - t_2) \mp (x_1 - x_2) - i\delta \text{sign}(t_1 - t_2)]^{-1}.$$

Thus, linear in Φ part of the phase in the $G_{R,L}(\Phi, ..)$ can be represent as

$$\int \frac{d\epsilon dp}{(2\pi)^2} \frac{\exp(ipx_1 - i\epsilon t_1) - \exp(ipx_2 - i\epsilon t_2)}{\epsilon \mp pv_f \pm i\delta \text{sign} p} \Phi(p, \epsilon).$$

So, Gaussian integral over the fields Φ can be easily calculated. As a result, one has the well-known expression for the short range interaction model:

$$G_{cul R}(x_1, t_1; x_2, t_2) = \frac{1}{2\pi i} [v_f^c(t_1 - t_2) - (x_1 - x_2) - i\delta \text{sign}(t_1 - t_2)]^{-1} [(\delta)^2/(x_1 - x_2)^2 - (v_f^c(t_1 - t_2) - i\delta)^2]^\alpha,$$

here α is $\sinh^2 \theta = (v_f^c - v_f)^2 / 4v_f^c v_f$ while θ is the diagonalizing angle has been introduced above.

Returning to the our question one can see from (B4) that effective interaction (the propagator of auxiliary boson field) equals:

$$V^{eff}(p) = V_0(p) \frac{\omega^2 - p^2 v_f^2}{\omega^2 - (pv_f^c)^2 + i\delta}. \quad (\text{B6})$$

In [17] the same result have been calculated for the short range interaction ($V_0(p) \rightarrow V_0$). It is important that the result was obtained from the equation for the effective interaction between I and J electrons, V_{IJ} :

$$V_{RR} = V_0 + V_0 \prod_R V_{RR} + V_0 \prod_L V_{LR}$$

$$V_{LR} = V_0 + V_0 \prod_R V_{RR} + V_0 \prod_L V_{LR},$$

where \prod_J is fully renormalized electron bubble. Yet, in [17] it was shown that the solution of the equations is the same as for the idem equations but with free electron bubble (consisting of free J -electron Green functions and without vertexes renormalization). It means that the RPA approach is exact for the Luttinger liquid. As it should be, V_{RR} coincides with our $V^{eff}(p)$.

Notice, the limitation of the Dzyaloshinskii - Larkin result for short range interaction is needless. As one can see from discussion given above (see Eqs.B4, B6) the RPA approach is valid for arbitrary $V_0(p)$. At the same time the short range interaction limitation is crucial for our Fermi liquid picture. A free fermion description is possible only at the length larger than characteristic size of an interaction. An attempt to generalize the proof of Appendix A for arbitrary interaction brings to four - fermions interaction is forbidden for the short range case. So, there is no direct relation between validity of the RPA approach and Fermi liquid description of the theory. One can give an opposite example. The result (B6) is valid so far as in the effective action the electron loops with a bigger number of the external fields Φ do not exist. The last depends not only of a model but and of a quantity calculated. For a Green function, the effective action for LL in terms of Φ -fields is free. Would one calculate a wave function (of a ground or an excited state) one has to use a functional integral defined on a set of the fields $\Psi(x, t)$ non-decaying in time (for a fermion theory, e.g. for Luttinger liquid, it was shown in [22]). It gives rise to an additional (exponential in Φ) terms in the action. As a result, calculation a functional integral (pairwise connections of the fields Φ) brings to

strong interaction in a boson system (with vertex renormalization, etc). It means that a RPA approximation will not exact for the case.

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- [1] J. Schwinger Phys. Rev. **128**, 2425 (1962).
 - [2] S. Tomonaga Prog. Theor. Phys. **5**, 544 (1950).
 - [3] J. M. Luttinger J. Math. Phys. **4**, 1154 (1963).
 - [4] D. C. Mattis and E. H. Lieb J. Math. Phys. **6**, 304 (1965).
 - [5] H. J. Schulz, G. Cuniberti, and P. Pieri, "Field Theories for Low Dimensional Condensed Matter Systems (Springer, Berlin, 2000)
 - [6] J. Voit Rep. Prog. Phys. **58**, 977 (1995).
 - [7] K.-V. Pham, M. Gabay, and P. Lederer Phys. Rev. B **61**, 1637 (2000)
 - [8] Abrikosov A.A., Gorkov L.P., Dzyaloshinski I.E. "Methods of quantum fields theory in statistical physics." (Prentice - Hall, Englewood Cliffs, 1963).
 - [9] L. D. Landau Sov. Phys. JETP **3**, 920 (1956)
 - [10] N. N. Bogolubov Nuovo cimento, **7**, 794 (1958), Zh. Eksp. Theor. Fiz. **38**, 58 (1958) and J. G. Valantin Nuovo cimento, **7**, 843 (1958).
 - [11] Yu. M. Galperin, V. L. Gurevich, and V. I. Kozub Sov. Phys. JETP **38**, 517 (1973) and A. G. Aronov, Yu. M. Galperin, V. L. Gurevich, and V. I. Kozub Advances in Physics **30**, 539 (1981).
 - [12] J. M. P. Carmelo and A. A. Ovchinnikov, J. Phys. Condens. Matter **3**, 757 (1991).
 - [13] J. M. P. Carmelo, P. Horsch Phys. Rev. Lett. **68**, 871 (1992)
 - [14] J. M. P. Carmelo, P. Horsch, P. A. Bares, and A. A. Ovchinnikov, Phys. Rev. B **44**, 9967 (1991);
 - [15] S. N. Artemenko and S. V. Remizov Phys. Rev. B **72**, 125118 (2005).
 - [16] J. M. P. Carmelo and A. H. Castro Neto, Phys. Rev. B **54**, 11 230 (1996).
 - [17] I. E. Dzyaloshinskii and A. I. Larkin, JETP **38**, 202 (1974).
 - [18] G. D. Mahan, "Many Particle Physics " (Plenum Press, New York, 1993).
 - [19] G. S. Danilov, I. T. Dyatlov, and V. Yu. Petrov, Zh. Eksp. Theor. Fiz. **79**, 2017 (1980) (Sov. Phys. JETP **52**, 1019 (1980)).
 - [20] F. D. M. Haldane, J. Phys. C **14**, 2585 (1981).
 - [21] There is direct relation between free fermion (in term χ - fields) and free boson representation of LL. Impossibility to reduce an initial electron Hamiltonian to the model with non-interacting bosons brings to impossibility formulate the Fermi liquid approach. It is well-known that the models with nonlinear electron dispersion (see, for example, the latest brief review A. Imambekov and L. I. Glazman, Science **323**, 228 (2009)) are the models with strong interaction between bosons. (See e.g. K. V. Samokhin, J. Phys. Condens. Matter **10** L533 (1998).) So, our approach cannot be applicable to such models directly.
 - [22] V. V. Afonin and V. Yu. Petrov, Zh. Eksp. Theor. Fiz. **134**, 637 (2008) (JETP **107**, 542 (2008)) and V. V. Afonin, V. L. Gurevich, and V. Yu. Petrov Zh. Eksp. Theor. Fiz., **135**, 969 (2009) (JETP **108**, 845 (2009)).
 - [23] G. Itzykson and J. Zuber "Quantum field theory" McGraw-Hill Book Company (1978).
 - [24] Hubbard J. Phys. Rev. Lett. **3**, 77 (1959).